Consider a cylindrical rod of length \( L \) and diameter \( d \) for carrying electrical current, \( I \). The rod is thermally insulated over its length except for the ends, which are maintained at two different constant temperatures (\( T_0 < T_1 \)). The current flows from high to low temperature (from \( T_1 \) to \( T_0 \)). The rod is made of a material that has constant thermal conductivity, \( k \) in W/m K, and electrical resistivity, \( \rho \) in \( \Omega \) m.

1. Write down the differential equation that can be used to describe the temperature profile in the rod. Treat as a 1-D problem with the \( x \) coordinate along the rod.
2. List the boundary conditions needed to solve the equation.
3. Solve the equation for the temperature profile along the rod. Sketch the normalized temperature profile \( \frac{T_1 - T(x)}{T_1 - T_0} \) for \( I = 0 \) and \( \rho l^2 = k \frac{A^2}{L} \).
4. Determine the heat load into the low temperature heat sink \( (Q(x = L)) \).
Problem 2

Consider a circular pipe flow as shown. For the first part of the derivation, one can simplify the problem by assuming that inside the pipe, the thermal distribution of the flow varies axially but can be considered uniform radially, i.e. $T=T(x)$. We can also assume the mass flow rate is constant, $\dot{m} = \rho AV$. The thermal conductivity of the fluid is a constant, $k$. The pipe is losing to the surrounding (with a convective constant $h$ to the surrounding temperature $T_\infty$). You can also assume that the pipe is thin walled.

a) Assuming steady state, derive a simple 2$^{nd}$ order differential equation for the temperature distribution along the pipe length, $T=T(x)$.

b) Solve the equation by assuming conduction is negligible and the inlet temperature is $T_O$ at $x=0$.

c) Assume uniform heat is suddenly generated ($u'''$-power per unit volume) within the pipe wall. Derive, respectively, equations for the temperature variation along the pipe wall, $T_p(x,t)$, and the flow temperature, $T(x,t)$. It might be convenient if you label properties of the pipe wall distinctly from properties of the fluid system. Eg. $A_P$ as the cross sectional area of the pipe wall, etc..

Briefly define each term.

Note: you do not need to solve these two equations.
A “synthetic” jet is shown in the figure above. In this device, a sinusoidal oscillating membrane, whose deflection is given by

\[ w(r,t) = \text{Re}\left\{ w_0 \left( 1 - \frac{2r}{D} \right)^2 e^{i\omega t} \right\}, \]

compresses and expands the fluid \( i = \sqrt{-1} \), angular frequency \( \omega \), speed of sound \( c \), density \( \rho \), dynamic viscosity \( \mu \), and thermal diffusivity \( \alpha \) in a circular cavity of diameter \( D \) and depth \( H \). As the cavity volume is decreased due to membrane motion, the piston compresses the fluid in the cavity and expels some fluid through a circular orifice of height \( h \) and diameter \( d \). Similarly, as the cavity volume is increased due to membrane motion, the fluid in the cavity expands and ingests some fluid through the orifice. **Note: drawing is NOT to scale.**

Find:

a) If the flow is considered to be incompressible, use control volume analysis to find an expression for the oscillatory mass flux (i.e., \( \dot{m}_{out}(t) \)).

b) If the circular orifice is sealed, what is the pressure oscillation in the cavity assuming isentropic conditions? What is the pressure oscillation in the cavity assuming isothermal conditions? Assume that that fluid is an ideal gas.

c) Physically describe the oscillatory flow in the orifice in terms of major losses and minor losses.

- What are the mechanisms for these losses? How would the ratio of these losses scale as \( d/h \) becomes very large or very small?
- Assuming laminar flow, describe the velocity profile in the orifice for very small \( d/h \) as a function of the Stokes’ parameter

\[ S = \sqrt{\frac{\omega d^2}{4\nu}}. \]

Physically, what does \( S \) represent the ratio of? In the limit of very small \( S \), what does this flow asymptote to? In the limit of very large \( S \), what does this flow asymptote to? Hint: this flow is similar to that of an oscillatory flow in a duct.
Problem 4

Consider a rigid stationary container of volume \( V \) with surface \( \partial V \) filled with a fluid of constant density \( \rho \) and constant viscosity \( \nu \). At \( t = t_0 \), the fluid inside the container is perturbed to have non-zero velocity \( \mathbf{u}(\mathbf{x}, t_0) \neq 0 \). For an viscous fluid, show that the velocity field always decays to zero. (Hint: consider \( \frac{d}{dt} \int_V \frac{\rho}{2} u_i u_i dV \))
Prove that:

a) the maximum thermal efficiency \( \eta_{th} \) of a heat engine, operated between a high temperature thermal reservoir at \( T_H \) and a low temperature reservoir at \( T_L \), is given by \( \eta_{th} = 1 - \frac{T_L}{T_H} \), where \( \eta_c \) is the thermal efficiency of a Carnot engine.

b) the coefficient of performance (COP) \( \beta' \) of a heat pump satisfies \( \beta' \leq \frac{T_H}{T_H - T_L} \).

c) If a heat engine is operated with a heat pump between the same thermal reservoirs and the heat engine produces work to feed the heat pump, discuss the overall effect of the combined system. Is the combined system a heat engine, a heat pump, or something else?
Figure 1 illustrates an example of a concentrated solar power system (CSP).

In general a CSP system is made of a concentrator, a receiver and a thermal power producing cycle. Figure 2 illustrates a simplified version of a CSP system, in which the power cycle is sandwiched between the receiver and the ambient.

In the United States, the Department of Energy, through the SunShot initiative has established that in order to make this power production strategy competitive, power cycles with thermal to mechanical efficiencies in excess of 50% are required.
Questions:

a. What is the minimum temperature at which the receiver should operate, if the SunShot efficiency goal is to be met? You can assume that the ambient is at standard temperature (20 °C).

b. Determine the receiver temperature that maximizes power output, if the power producing space sandwiched between, $T_r$, and the ambient does not generate any entropy, the heat transfer rate from the concentrator, $\dot{Q}_c$, is fixed and the receiver experiences losses (by convection and radiation) that can be described by a function, $f$, of the receiver temperature, $T_r$,

$$\dot{Q}_{loss,\text{receiver}} = f(T_r)$$

Clearly list all your assumptions and JUSTIFY them.