Problem.

(a) Construct a transfer function $G(s)$ that has infinite gain margin and infinite phase margin and sketch (general features only) its Nyquist diagram.

(b) Construct a transfer function $G(s)$ that has infinite gain margin and finite (positive) phase margin and sketch (general features only) its Nyquist diagram.

(c) Sketch the Nyquist diagram of a transfer function $G(s)$ that has finite gain margin and negative phase margin. You do not have to state an exact transfer function $G(s)$ that has the features in the Nyquist diagram you sketch. However, describe some of the dominant characteristics of $G(s)$ in terms of:

1) number and type of poles,
2) number and type of zeros,
3) relative relationship between the poles and zeros,
4) and whatever else you think is important.

State briefly how you would go about constructing a $G(s)$ that has the features of your Nyquist diagram.
Problem. The bilinear rule is given by
\[ z = \frac{1 + Ts/2}{1 - Ts/2}, \]
whereas the backward rectangular rule is given by
\[ z = \frac{1}{1 - Ts}. \]
These two functions map the closed left-half s-plane to one of the following, closed shaded regions of the z-plane:

Note that the large circles in the above figures are the unit circle.

(a) Match the bilinear rule and backward rectangular rule with the appropriate closed, shaded region in the z-plane and prove that this region is the result of mapping the closed left-half s-plane. Do not simply show that the \( j\omega \) plane maps to the outside of the appropriate shaded region; show that the entire closed left-half s-plane maps to the shaded region.

(b) If the above rules are used to find a \( D(z) \) that approximates \( D(s) \), which rule would you prefer and why? Will the preferred rule be guaranteed to be better than the alternative rule? Why or why not? Be brief in your answers.
Pretend that you work at a car assembly company. You have been given the below data plot by your supervisor. It represents the speed response for a medium speed torque motor like you may find on some electric cars and ships. She tells you to use a Proportional-Integral (PI) controller with the transfer function \( C(S) = K \frac{S+10}{S} \) to design a closed loop cruise control system for the application to an electric car. Your supervisor further explains to you that \( K \) is a variable, which she intends technicians to tune at a later date as cars finish assembly at your company.

Your division has a requirement that closed loop controllers must provide a minimum of 6 dB of gain margin and 30 degrees of phase margin on all closed loop applications.

Considering the possible range of final values for \( K \) that any technician may choose, is your supervisor’s choice of controller a good choice? Explain your answer in detail. Give any limits on \( K \) that you might recommend.
Two wheels (mass m) are connected by an axle (length L) and roll without slipping. Assume that all mass is concentrated at the center of each wheel. The coordinates of the center of mass (c.m.) for the systems is (x,y) and the angle $\phi$ measures the orientation of the axle from the horizontal. Initially the system is at the origin moving with velocity $v_0$ in the direction of the positive x-axis. Also, $\omega(0) = 0$, and $\phi(0) = 0$.

1) Express the nonholonomic constraint equation as a differential equation.
2) Write the Lagrangian function for the system.
3) Obtain the differential equations of motion for the system.
4) Prove that the constraint force is equal to $-2m \omega v_0$.
5) Describe the path x,y that the center of mass follows.
In the system above, a double pendulum is attached to the ground via a torsional spring with a stiffness of $K$ with a set point at $\theta = 0$. The first and second links, of lengths $l_1$ and $l_2$, respectively, are attached via a pin joint. In addition, a rotational damper with a damping coefficient of $b$ is between these links. The mass of the links is negligible, though a mass $m$ hangs at the end of the second link. A force also acts on the system at the joint between the two links and is always perpendicular to the first link.

Given this system and assuming that $F = 0$, at what values of $\theta$ is it possible to have an equilibrium point assuming the spring cannot rotate counter-clockwise (i.e. $\theta$ cannot be less than 0)?

Find all values of $K$ for which an equilibrium point exists with link 1 oriented horizontally when $F > 0$.

Derive the equations of motion for the system in terms of the state variables $\theta$ and $\phi$. 
The solid circular disk of mass $m$ and small thickness is spinning freely on its shaft at the rate $\dot{\phi}$. If the assembly is released in the vertical position at $\theta = 0$ with $\dot{\theta} = 0$,

a) Determine the rate $\dot{\theta}$ as the position $\theta = \frac{\pi}{2}$ is passed.

b) Determine the horizontal components of the forces A and B exerted by the respective bearings on the horizontal shaft as the position $\theta = \frac{\pi}{2}$ is passed.

Note: Neglect the mass of the two shafts compared with $m$, and neglect friction.