Problem 1
Consider uniform flow over a sphere with diameter, d. The uniform flow has velocity U with constant dynamic viscosity \( \mu \) and constant density \( \rho \). Here, we define the drag coefficient to be

\[
C_D = \frac{D}{\frac{1}{2}\rho U^2 A}
\]

where D is the drag on the sphere and A is the cross-sectional area of the sphere.

a) Illustrate how the drag coefficient varies for Reynolds number over the range of \( 10^{-1} \) to \( 10^6 \). Annotate your plot and identify key features.

b) Using dimensional and/or physical arguments, for flows with \( \text{Re} \ll 1 \), find the relationship between \( C_D \) and \( \text{Re} \) (i.e., what is \( k \) in \( C_D \propto \text{Re}^k \) ?).

c) In a similar manner, find the relationship between \( C_D \) and \( \text{Re} \) for flows with \( \text{Re} \gg 1 \). Assume the Reynolds number is less than \( 10^5 \).

d) Sketch the streamlines for flow conditions in sub-questions b) and c). Briefly discuss the role of boundary layer separation, if any, for \( \text{Re} \ll 1 \) case.

e) Suppose there is a perfectly spherical raindrop. Find its terminal velocity in m/s. Assume that the raindrop does not deform and has a diameter of 2mm. The values for dynamic viscosity of air and water are \( 1.8 \times 10^{-5} \text{Ns/m}^2 \) and \( 1.0 \times 10^{-3} \text{Ns/m}^2 \), respectively. How would you verify your solution using the figure you drew in sub-question a).
Problem 2
The figure below shows steady, incompressible, laminar flow at the entrance to a pipe of radius $a$. Assume that the entrance to the pipe is well-rounded such that the inlet velocity profile is uniform $u(r, x = 0) = U$. A fixed, non-deformable control volume is shown.

![Diagram of flow through a pipe](image)

a) Provide a sketch of the velocity profile at 2 locations: $x < x_L$ and $x \geq x_L$, where $x_L$ is the entrance length. Explain your figure.

b) Write an expression for the well-known parabolic velocity profile $u(r)$ in terms of the centerline velocity $u_0$ in the fully developed region. Is this profile valid for $x < x_L$?

c) Write the integral continuity equation and simplify it for this flow. In the fully-developed region, determine the ratio of the centerline velocity to the uniform inlet velocity $u_0/U$.

d) Determine the shear stress in the fully-developed flow region $\tau_{fd}$.

e) Write the integral axial momentum equation and simplify it for this flow. Show that the dimensionless pressure drop from $x = 0$ to $x > x_L$ is $\frac{p_0 - p(x)}{\frac{1}{2} \rho U^2} = 2 \int_0^x \frac{\tau(x)}{\frac{1}{2} \rho U^2} dx + \frac{2}{3}$.

What is the physical interpretation of the 2/3 term?

f) Add and subtract $\tau_{fd}$ to $\tau(x)$ and simplify to show that

$$\frac{p_0 - p(x)}{\frac{1}{2} \rho U^2} = 2 \int_0^x \frac{\tau(x) - \tau_{fd}}{\frac{1}{2} \rho U^2} dx + \frac{2}{3} + \frac{64}{14 \cdot 4 \cdot 24} \frac{x}{\text{Re} \cdot \frac{D}{\mu}}$$

What is the physical relevance of terms I and II?
Problem 3
A very long, circular metal rod (with a length of \( L \), a diameter of \( D \), and the thermal conductivity of \( k \)) is connected to an electronic device as shown to enhance heat dissipation. In order to operate the device properly, the base temperature of the rod is required to be maintained at a constant \( T_0 \). This can be achieved by blowing a constant stream of air across the metal rod (with a velocity \( U_\infty \) and a temperature of \( T_\infty \), as shown).

a) Determine the temperature distribution of the rod, \( T(x) \), assuming the origin is located at the center of the rod.

b) Determine the heat dissipated from the device.

c) Assuming the convective heat transfer relation follows the empirical formula:
   \[ Nu = C Re^{1/2} Pr^{1/3} \]
   where \( Nu \), \( Re \), \( Pr \) are Nusselt number, Reynolds number and Prandtl number, respectively, and \( C \) is an empirical constant, determine which of the following two options is more effective to dissipate heat from the device:
   - **option 1** is to double the free stream velocity and
   - **option 2** is to increase the diameter of the cylindrical rod by 20%.

*Note:* Feel free to make the necessary assumptions to solve this problem, clearly stating and justifying your assumptions. Also assume that these devices are well insulated everywhere and only heat loss will be through the metal rod.
Problem 4

A current lead consists of a solid rod with internal heat generation \( q = \frac{\rho I^2}{A^2} \) where \( \rho \) is the electrical resistivity of the rod material, \( I \) is the transport current and \( A \) is the cross section of the rod. The rod is thermally insulated on its side, so that the only heat transfer is by conduction to the two end reservoirs at \( T_H \) and \( T_c \). Assume that \( T_H > T_c \) and the thermal conductivity, \( k \), and resistivity, \( \rho \), are constant.

\[ \begin{array}{c}
\text{T}_H \\
\text{L} \\
\text{x} \\
\text{T}_C 
\end{array} \]

a) Derive a differential equation to describe the temperature profile along the rod.
b) List the appropriate boundary conditions (this is a steady state problem).
c) Solve the differential equation for the temperature profile and sketch below approximate \( T(x) \) for \( q = 0 \) and two values of \( q > 0 \).
d) Such a lead is optimized by minimizing the heat transfer to the cold reservoir for a given current, \( Q_L \). Write an expression for \( Q_L \) and find the condition for \( A \) and \( L \) corresponding to the minimum.
Problem 5

The entropy generation number, $N_s$, for a balanced counterflow heat exchanger, in the limit of negligible pressure drop is given by:

$$N_s = \frac{\dot{S}_{gen}}{\dot{m}c_p} = \ln \left( \frac{1 + \frac{T_1}{T_2} N_{tu}}{1 + N_{tu}} \right) \left( 1 + \frac{T_2}{T_1} N_{tu} \right) \left( 1 + N_{tu} \right)$$

This behavior is illustrated in the figure.

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a. (20%) Explain the physical reason behind the observed behavior of the entropy generation number with the variation of the heat exchanger $N_{tu}$.  

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Problem 5, cont’d

b. (10%) The entropy generation number exhibits a maximum with respect to \( N_{tu} \). Find an expression for the maximum entropy generation number.

c. (70%) Proof that the maximum entropy generation number (with respect to number of heat transfer units) in a balanced counterflow heat exchanger in the limit of zero-pressure drop irreversibility coincides with the entropy generation number found for an adiabatic chamber in which two streams, \((\dot{m}, T_1)\) and \((\dot{m}, T_2)\), carrying the same fluid, are being mixed steadily at constant pressure.
Problem 6

The power plant illustrated in the figure contains three compartments: the hot-end heat exchanger, an inner compartment that houses the circulating working fluid, and the cold-end heat exchanger. Assume that the heat transfer rates are proportional to the respective temperature differences,

\[ \dot{Q}_1 = \frac{1}{R}(T_1 - T_{1C}) \quad \dot{Q}_2 = \frac{1}{R}(T_{2C} - T_2) \]

and that the thermal resistance \( R \) characterize both heat exchangers. Assume also that \( \dot{Q}_1 \) and \( \dot{Q}_2 \) can be varied and that the inner compartment operates reversibly.

![Diagram of power plant with heat exchangers and temperature notation]

a. Derive an expression for the instantaneous power output \( \dot{W} \) as a function of \( T_{1C}, T_1, T_2, \) and \( R \).
Problem 6, cont’d

b. Maximize $\dot{W}$ with respect to $T_{1C}$ and show that $T_{1C}/T_{2C} = (T_1/T_2)^{1/2}$.

c. Show that the efficiency $\eta = \dot{W}/\dot{Q}$ at maximum power is equal to $1 - (T_2/T_1)^{1/2}$

d. Calculate the ratio of the entropy generated in the hot heat exchanger divided by the entropy generated in the cold heat exchanger, and show that the cold heat exchanger generates more entropy.