**Problem.** A compensator $D(s)$ is given by

$$D(s) = \frac{s^2 + 2^3 z! n s + !^2_n}{s^2 + 2^3 p! n s + !^2_n}.$$ 

(a) What is the magnitude and phase of $D(j\omega)$ for $\omega=\omega_n$?

(b) Show that $|D(j\omega)|$ has a local minimum or maximum at $\omega=\omega_n$. (Hint: First, find an expression for $|D(j\omega)|^2$.)

(c) Sketch $|D(j\omega)|$ for the following three cases:

i. $\zeta_z = \zeta_p$

ii. $\zeta_z < \zeta_p$

iii. $\zeta_z > \zeta_p$
Problem. Consider the plant

\[ G(s) = \frac{2}{(s + 1 + j)(s + 1 - j)(s - 1)} \]

under a compensator of the form,

\[ D(s) = K \frac{s + a}{s + b} \]

(a) If \( D(s) \) is restricted to be a **lead compensator**, use root locus concepts to determine how one (qualitatively) can choose \( a \) and \( b \) to help ensure there exists \( K_{cr,1} \) and \( K_{cr,2} \) satisfying \( K_{cr,1} < K < K_{cr,2} \leq \infty \) such that the closed-loop system is stable for \( K_{cr,1} < K < K_{cr,2} \)? If so, is \( K_{cr,2} < \infty \) or does \( K_{cr,2} = \infty \).

(b) If \( D(s) \) is restricted to be a **lag compensator**, without computing angles of departure or real-axis break-away points, sketch two possible root loci. Comment on how difficult it would be to find a lag compensator that results in a stable closed-loop system.

**YOU MUST SHOW HOW YOU DRAW YOUR CONCLUSIONS FOR FULL OR PARTIAL CREDIT.**
Question.

You are given the open loop system,

\[ G(s) = \frac{K(s+0.4)}{s^2(s+3.6)}. \]

(a) Sketch the root locus for the system.

(b) To give the system a stable response with a little overshoot and zero steady state error, you are told by your boss that you can apply a PID controller of the form,

\[ G(s) = k_p + k_d s + \frac{k_i}{s}, \]

where \( k_p, k_d, \) and \( k_i \) are positive non-zero constants, to the system. Is your boss correct? Justify your answer.
1. Write the equations of motion for the system in terms of the center of the disc. Assume that there is no friction between the collar and the rod and that the disc rolls without slipping along the surface.

2. What is the velocity of the collar when it impacts the ground? Assume the following parameter values: $m = 1\text{kg}$, $M = 10\text{kg}$, $L = 2\text{m}$, $R = 1\text{m}$, $\theta(0) = 60^\circ$, and $\dot{\theta}(0) = 60^\circ/\text{sec}$.

3. Assume that the disc is instead a sphere with a velocity coming out of the page of $2\text{m/s}$. What is the velocity of the collar when it impacts the ground for this case? Assume the previous parameter values.
Each of the two circular disks has a mass $m$ and is welded to the end of the rigid rod of mass $m_0$ so that the disks have a common $z$-axis and are separated by a distance $b$. A couple $M$, applied to one of the disks with the assembly initially at rest, gives the centers of the disks an acceleration $\mathbf{a} = -a\mathbf{i}$. Friction is sufficient to prevent slipping. Derive the expressions for the normal forces $N_A$ and $N_B$ exerted by the horizontal surface on the disks as they begin to roll. Express the results in terms of the acceleration $a$ rather than the moment $M$. 

![Diagram of two disks with a rigid rod and a couple force applied to one of the disks](image-url)
A rocket of mass $M$ and moment of inertia $I$ is constrained to move in a horizontal plane. Its thrust is directed such that its velocity is always directed toward a target which moves along the $x$-axis according to $x_{\text{target}} = a \sin \omega t$. A moment about the center of mass controls the orientation of the craft.

a) Find the constraint equation on the velocity.
b) Find the constraint equation on the orientation of the rocket.
c) Find the coefficients of the Lagrange multipliers.
d) Find the generalized forces associated with the coordinates $x, y, \theta$ for the force $F$ and moment $M$ and the virtual work done by them.
e) Find the equations of motion.