**Question.** Consider the unity feedback system shown below.

\[ G(s) = \frac{(s - 1)[s + (0.1 - j2)][s + (0.1 + j2)]}{(s + 0.1)[s + (0.1 - j1)][s + (0.1 - j2)]}. \]

Assume \( H(s) = K \) and sketch the root locus.

b) Repeat (a) assuming

\[ G(s) = \frac{(s - 1)[s + (0.1 - j2)][s + (0.1 + j2)]}{(s + 10)[s + (0.1 - j1)][s + (0.1 - j2)]}. \]

c) Assuming perfect knowledge of the plant \( G(s) \), can you find a compensator of the form \( H(s) = KD(s) \) such that the feedback system is stable for all \( K \)? If so, discuss whether it makes sense to implement this compensator on a real system.
**Question.** Recall that assuming a zero order hold, the continuous-time state space system,

\[
\dot{x} = Fx + Gu \\
y = Hx + Ju
\]  

has the exact discrete-time representation,

\[
\begin{align*}
x(k+1) &= \Phi \dot{x}(k) + iu(k) \\
y(k) &= \Gamma \dot{x}(k) + Ju(k)
\end{align*}
\]  

where

\[
\begin{align*}
\Phi &= e^{FT} \\
i &= (\begin{pmatrix} 1 & 0 \\ 0 & e^{F_T} \end{pmatrix})G
\end{align*}
\]  

a) As \( T \to 0 \), what happens to \( \Phi \) and the eigenvalues of (2)? Also, what happens to \( \Gamma \)? (Hint: Recall that \( e^A = I + (1/2)A^2 + (1/3!)A^3 + \ldots \))

Now, define the delta operator,

\[
\pm \dot{x}(t) = \begin{cases} \\
\frac{d}{dt} \dot{x}(t) & \text{for } T = 0 \\
\dot{x}(t+T) - \dot{x}(t) & \text{for } T \neq 0
\end{cases}
\]  

We may then express (2) as

\[
\begin{align*}
\pm \dot{x}(t) &= \Phi \pm \dot{x}(t) + i \pm u(t) \\
y(t) &= \Gamma \pm \dot{x}(t) + Ju(t)
\end{align*}
\]  

b) Express \( \Phi_\delta \) and \( \Gamma_\delta \) in terms of the matrices appearing in (2). (Show how you arrive at your answer.)

c) \( \Phi_\delta \) and \( \Gamma_\delta \) may also be expressed as \( \Phi_\delta = \Pi(F,T)F \) and \( \Gamma_\delta = \Pi(F,T)G \). What is the infinite series for \( \Pi(F,T) \)? (Hint: Recall that \( e^A = I + (1/2)A^2 + (1/3!)A^3 + \ldots \))

d) As \( T \to 0 \), what happens to \( \Phi_\delta \) and the eigenvalues of (5)? Also, what happens to \( \Gamma_\delta \)?
**Question.** Consider the following unity feedback system.

![Feedback System Diagram](image)

Suppose, the Nyquist plot of $G(s)$ is given by the following:

Nyquist Diagram

- **a)** Determine the number of poles and zeros and whether they are located in the right half plane, left half plane or on the imaginary axis.

- **b)** Discuss stability of the feedback system as a function of $K$. 

**Question**

Consider the dynamic system depicted below:

This system consists of 3 shafts A, B, and C connected by gears. The gears are in a plane and mesh perfectly. Each shaft/gear is rigidly connected and has its own rotational inertia, $J_A$, $J_B$, and $J_C$, respectively. An x,y,z coordinate frame is defined for each shaft/gear, and the only motion permitted each shaft is rotation about their own z-axis. The radius of gear A is $r_A$, the radius of gear B is $r_B$, and the radius of gear C is $r_C$. Gear/Shaft A and C have linear viscous rotational dampers $B_1$ and $B_3$ acting on their shafts. Gear B has a constant magnitude externally applied torque ($T$).

Find the equation of motion governing shaft A as a function of time and express it in terms of $\omega_A$ and the constants described above.

Describe the assumptions that you make, show your work, and describe all possible forms of the solution that you consider.
Two carts of mass $M_0$ and $M_1$, connected via a spring and a damper, move on a horizontal surface without frictional losses. The cart of mass $M_1$ has a circular cylindrical surface of radius $R$ on which a small disk of mass $m$ and radius $r$ rolls without slip. Also, the cart of mass $M_1$ is acted upon by a horizontal force $F(t)$. Assume gravity acts in the vertical direction.

a) Draw the free body diagram of the cart of mass $M_0$ and find its equation of motion.

b) Find the velocity of the center of the disk of mass $m$.

c) Find the angular velocity of the disk of mass $m$.

d) Find the kinetic and potential energies of the disk.

e) If the system is released from rest in the configuration shown with $F(t) = 0$, what would you expect to happen?

f) If $F(t)$ takes the form $A\sin \omega t$, what would you expect as $\omega$ is increased?

g) Draw the free body diagrams of the disk and the cart of mass $M_1$.

h) Find the equations of motion of the cart and disk.

i) What changes about the solution technique if we assume the disk is both rolling and sliding on the cylindrical surface?

![Diagram of the system]
Question

Consider the dynamic system depicted below:

![Diagram of a long thin object with rounded ends of length (L). It is held stationary above the ground at a height (H) (H ≤ L) and at an angle (θ). It is then released and allowed to fall to the ground. The object has a mass (m), a rotational moment of inertia in the plane of the page (I). It is made of a homogeneous material and the center of mass is at the geometric center at an initial horizontal position x = x₀ = 0.]

This system consists of a long thin object with rounded ends of length (L). It is held stationary above the ground at a height (H) (H ≤ L) and at an angle (θ). It is then released and allowed to fall to the ground. The object has a mass (m), a rotational moment of inertia in the plane of the page (I). It is made of a homogeneous material and the center of mass is at the geometric center at an initial horizontal position x = x₀ = 0.

After the object falls and come to rest what will be the final horizontal position of the center of mass?

How does this position depend on the effective coefficient of restitution between the object and the ground (C_R), the coefficient of friction between the object and ground (μ) (you can assume the the dynamic and static coefficients of friction are the same), and the initial height of the COM (H)?

Describe the assumptions that you make, show your work, and describe all possible forms of the solution that you consider.