Prelim Exams 2009: Materials

#1
(O.E.)

a) What is meant by the term unit cell as applied to a metal structure?
b) What is meant by the term coordination number of an atom in a metal structure? What is the coordination number of BCC and FCC crystal structures?
c) Draw the unit cell of lithium (BCC structure) and use it to explain why this unit cell contains the equivalent of 2 lithium atoms.
d) Silicon takes a diamond cubic (DC) structure. How is this structure related to an FCC structure?
e) The perovskite crystal structure, which is a common structure of ferroelectric oxides like BaTiO₃, is usually drawn in a cubic cell with Ti atoms at the corners, Ba at the center and O at centers of each of the edges. Draw the structure and show that it has the stoichiometric formula BaTiO₃.

Crystal structure and atomic structure for metals

<table>
<thead>
<tr>
<th>Metal</th>
<th>Crystal structure</th>
<th>Atomic radius (nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cobalt</td>
<td>HCP</td>
<td>0.125</td>
</tr>
<tr>
<td>Silver</td>
<td>FCC</td>
<td>0.144</td>
</tr>
<tr>
<td>Lithium</td>
<td>BCC</td>
<td>0.152</td>
</tr>
</tbody>
</table>

f) Lithium has a much lower density than the other metals in the table above. Considering material properties and crystal structure, provide reasons to explain this behavior.
For a tensile test, it can be demonstrated that necking begins when

\[
\frac{d\sigma_T}{d\varepsilon_T} = \sigma_T
\]

(a) Using \( \sigma_T = K\varepsilon_T^n \) determine the value of the true strain at the onset of necking.

(b) A polycrystalline metal has a plastic true stress-strain as described by the Hollomon’s relationship above. Determine \( n \) knowing that the flow stresses of this material at 0.1 and 0.2 % plastic deformation are equal to 175 and 185 MPa, respectively.

(c) What is the ultimate tensile strength of the material in (b)?

(d) Consider a cylinder specimen of some hypothetical metal alloy that has a diameter of 10.0 mm. A tensile force of 1500 N produces an elastic reduction in diameter of \( 6.7 \times 10^{-4} \) mm. Compute the modulus of elasticity for this alloy, given that the Poisson’s ratio is 0.35.
Consider the figure below of two parallel edge dislocations of opposite sign, located on parallel slip planes.

(a) Find the interaction force between the two dislocations
(b) Determine the stable glide equilibrium position(s) for this pair of dislocations
(c) If diffusion can occur, how will dislocation II move relative to dislocation I?
(i) What are the underlying macroscopic and microscopic events that occur at fracture?

(ii) In two steels that fracture after elongations of 0.5% and 50%, what is the essential difference in the deformation processes occurring in them?

(iii) Sketch likely stress-strain curves for the above two steels, assuming that they are identical composition steels, one of which has been left in the softest condition and one which has been hardened to the maximum possible.

(iv) A machine tool part fabricated from Al alloy fractures at a stress of 300 MPa for cracks of maximum internal length 4 mm. Now it is desired to use the same part at the lower stress of 260 MPa but to allow a greater internal crack length of 6 mm, since detecting cracks of less than 5 mm is not possible with present crack detection equipment. Given that $K_{ic} = 40 \text{ MPa}\sqrt{\text{m}}$, calculate whether the proposed use is safe or not.
(i) Look at the Cu-Zn and Pb-Sn diagrams and mark on them clearly a peritectic, a eutectic and a eutectoid reaction. Write out the reactions below that correspond to your choices.

(ii) What is the maximum solubility of Pb in Sn and Sn in Pb in solid and liquid phases?

(iii) If I want to solder using the minimum freezing range, what composition should I pick? If I want to allow a freezing range of about 30° C, what composition(s) should I use?

(iv) What are the weight percentages of the phases in the Pb-40wt% Sn alloy at 300, 200 and 100°C, assuming that they are at equilibrium in each case?

(4 points) Assuming that I cooled the alloy quite fast how might your answer change?
Lead - Tin phase diagram.

Copper - Zinc phase diagram.
Prelim Exams 2009: Mechanics

#1
(w.o.)

Figure 1 shows an elastic beam of length L, modulus E, and moment of inertia I which is supported by a spring with stiffness k at point B. The force in the spring is zero when the bar is in the horizontal position. (a) Find the vertical displacement, $u_C$ of the end C due to the magnetic body force ($F_m$). (b) If the yield stress is $\sigma_0$, what is the magnetic body force required to plastically deform the bar?

![Diagram of cantilever beam with spring and magnetic force](image)

Figure 1: Cantilever supported by a spring with spring constant k (N/m). The body force $F_m$ is normalized per unit length (N/m).
A passenger side airbag inflator is illustrated below. During a head-on collision, an accelerometer triggers the combustion material within the inflator. This creates large dynamic loading with maximum pressure of 8 MPa lasting 10 msec. The mass flux coming out of the inflator also creates a simultaneous, constant bending moment of $M = 40$ N-m along the $z$ axis that also last 10 msec. If the material used is steel with a perfectly plastic constitutive relation, as given in the Figure 2, find the final circumferential strain and longitudinal strain after the combustion process has completed. State your assumptions.

![Diagram of airbag inflator and stress-strain curve](image)

- outer radius, $r = 5$ cm
- wall thickness, $t = 1$ mm
- modulus, $E = 50$ GPa

Figure 2: (a) Airbag inflator that is constrained on both ends. (b) Perfectly plastic stress strain constitutive relation.
A block of material is stressed in the x- and y-direction as shown, but rigid walls prevent deformation in the z-direction. The ratio of the two applied stresses is constant, so that $\sigma_y = \lambda \sigma_x$. Answer the following by deriving equations expressed in terms of $\sigma_x$, $\lambda$, and the elastic constants of the material:

(i) Does a stress develop in the z-direction? If so, how is it related to $\sigma_x$, and the other constants involved?

(ii) Determine the stiffness $E' = \sigma_x / \varepsilon_x$ for the x-direction.

(iii) Compare this apparent modulus $E'$ with the elastic modulus $E$ from a uniaxial test. (Suggestion: Consider $\lambda$ values of -1, 0 and 1 and assume $\nu = 0.3$.)

(iv) Will the stress at the rigid walls be different if the material was heated instead of being subjected to the stresses shown?
Superposition is a commonly used technique in many areas of science and engineering. Consider a hollow steel cylindrical shaft, outer diameter do, inner diameter di and length L, fixed to a rigid wall at one end and subjected to a torque T, axial force F (tensile), and bending moment M. Discuss the stress and strain state as a function of location in the shaft (in the cross-section and along the length). Discuss the applicability of superposition to this shaft – can it be used? Under what conditions is it applicable? State and assumptions you make and explain why you have made them. Would your answers be different if the shaft was made of plastic? Wood? Explain. Would the strain state differ significantly if the cross-section were a rectangle rather than a circle? Explain.
Two forces are applied to the component as shown. Knowing that the cross-section is a 30x45-mm rectangle, determine the normal and shearing stresses at points \( a \), \( b \), and \( c \). Determine the principal stresses and the maximum shear stress at \( a \) and \( b \).

If a torque \( T \) about the z axis through point A were also applied, discuss how that would affect the analysis of the stress and strain state of the component.